Brahmagupta

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Brahmagupta's life

Brahmagupta was born in 597 in the city of Bhinmal in the state of Rajasthan in Northeast India and lived by 668 at the age of 71. He was born in a mathematical family, as his father was an astrologer. In fact, Brahmagupta considered himself to be an astrologer like his father rather than a mathematician. He relied more on his religion to be his guidance in his discoveries. Many of his proofs and claims of the world in mathematics are written as poetry, yet theyare still accounted as proofs and claims.

Into adulthood, Brahmagupta became the director of the astronomical observatory placed in Ujjain, India. From there on he published four novels on astronomy and mathematics. diurnal

Brahmagupta's mathematical works

His works included astronomy, gravity theory, negative numbers, use of zero, quadratic equations and square roots.Most of the information published by Brahmagupta is found in ancient texts, such as the *Brahmasputasiddhanta* , twenty five chapters long. The meaning behind *Brahmasputasiddhanta* is "the system of the god of creation and astronomy". Within the first ten chapters of the book, Brahmagupta wrote things common to the mathematical and astronomical developments at the time. The had topics on mean longitudes of planets, true longitudes of planets, diurnal rotation, lunar eclipses, solar eclipses, risings and settings of the sun/moon, the phases of the moon, and the conjunctions of planets with the stars and other planets. The second half of the book holds major insight of the world. It holds explanations of the first half, algebra, observations and calculations on the calendar, meters, spheres, and instruments.

One well known explanation of algebra Brahmagupta explained in *Brahmasputasiddhanta* was multiplication using a place-vale system. For example, take the product of 354 multiplied by 593, and set up the multiplication as shown.

5 354

9 354

3 354

—————-

Then multiply the top row together where 5 is multiplied by every individual integer in 354. For example $5x4 = 20$. Therefore, put the 0 under the 4 and add the 2 to the next value and so on, moving each following product to the right one more place than the last.

5 354

9 354

3 354

—————-

1770

31860

1062

Then add these numbers together.

5 354

9 354

3 354

—————-

1770

31860

1062

—————-

209922

His other works were the *Cadamekela* , the *Khandakhadyaka* , and the *Durkeamynarda* which consists of works written in only verses and no proofs. I think is most famous and well known contribution to the mathematics world is his definition and use of zero with negative numbers. The truths of the universe could not have been discovered, written, and understood by mathematicians without Brahmagupta's concept of zero.

He stated: *When zero is added to a number from a number, the number remains unchanged; and a number multiplied by zero becomes zero.*

He did create a set of rules for zero as follows:

A debt minus zero is a debt.

A fortune minus zero is a fortune.

Zero minus zero is a zero.

A debt subtracted from zero is a fortune.

A fortune subtracted from zero is a debt.

The product of zero multiplied by a debt or fortune is zero.

The product of zero multiplied by zero is zero.

The product or quotient of two fortunes is one fortune.

The product or quotient of two debts is one fortune.

The product or quotient of a debt and a fortune is a debt.

The product or quotient of a fortune and a debt is a debt.

These simplified rules in different terms are the rules for zero we use today. In other words, Brahmagupta's rules for zero is as follows:

A positive number minus zero is the same positive number.

A negative number minus zero is the same negative number.

Zero minus zero is zero.

A negative number subtracted from zero is its opposite value (a positive number).

A positive number subtracted from zero is its opposite value (a negative number).

A number, positive or negative, multiplied by zero is zero.

Zero multiplied by zero is zero.

The product or quotient of two positive numbers is a positive number.

The product of quotient of two negative numbers is a positive number.

The product or quotient of a positive and negative number, or vice versa, is a negative number.

Some of his rules of zero did end up being incorrect, such as the fact that zero divided by zero is zero.

Brahmagupta also worked on the rules and solutions for arithmetic sequences, quadratic equations with real roots, infinity, and contributed to the works of Pell's Equation. Brahmagupta was fascinated in arithmetic equations and gives the formulas for finding the sum of squares and cubes to the nth integer. He also computed an algorithm for square roots equivalent to the Newton-Raphson Iterative Formula.

Brahmagupta's Algorithm for Square Roots

$$x^{\frac{1}{2}} = (x - y)(x + y) + y^{\frac{1}{2}}$$

The Newton-Raphson Iteration

$$x_{n+1} = x_1 - \frac{f(x_n)}{f'(x_n)}$$

Another famous work of his was finding the formulas for the area of a triangle and cyclic quadrilateral in terms of sides.

Area of a Triangle

$\sqrt{s}(s-a)(s-b)(s-c)$

where s is the semiperimeter and a, b, c are the sides of the triangle

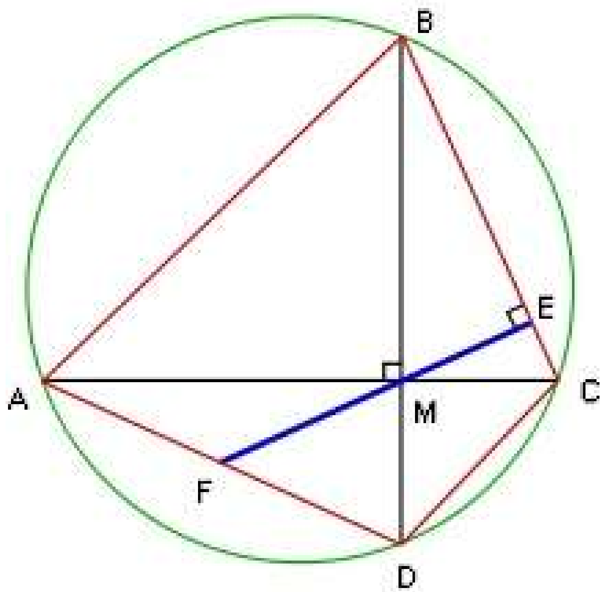
Area of a Cyclic Quadrilateral

$\sqrt{(}s(s-a)(s-b)(s-c)(s-d))$

where s is the semiperimeter and a, b, c, d are the sides of the quadrilateral.

The semiperimeter is equal to the sum of the number of sides of the figure divided by two.

From finding the area of a cyclic quadrilateral, Brahmagupta's Thereom was established.



Furthermore, Brahmagupta solved quadratic indeterminate equations:

$ax^2 + C = y^2$ and $ax^2 - c = y^2$

One equation he solves was $61x^2+1 = y^2$ which was astonishing because its smallest solution was $x = 226153980$ and $y = 1766319049$.

Moreover, Brahmagupta gives the solution to the quadratic equation.

$$x = \frac{-b+-\sqrt{b^2-4ac}}{2a}$$

Other works by Brahmagupta, other than the extensive and lengthy work on roots, geometry, and astrology, included work that applies to the Pell's equation. Brahmagupta solved the Pell's equation

$$nx^2 + 1 = y^2$$

using a poetic story format to express his mathematical findings like always. In modern notation, however, the solution Brahmagupta gave was

$$x = \frac{2t}{t^2 - n}$$

and

$$y = \frac{t^2 + n}{t^2 - n}$$

where t can be replaced by any number.

For example, if $t = 4$, the the solution comes to be

$$x = \frac{8}{16 - n}$$

and

$$y = \frac{16 + n}{16 - n}$$

Therefore,

$$nx^2 + 1 = n(\frac{8}{16 - n})^2 + 1$$

$$= \frac{64n}{256 - 32n + n^2} + 1$$

$$= \frac{64n + 256 - 32n + n^2}{256 - 32n + n^2}$$

$$= \frac{256 + 32n + n^2}{256 - 32n + n^2}$$

$$= (\frac{16 + n}{16 - n})^2 = y^2$$

Brahmagupta only gave the general solution because t can be replaced with any number.

Collaboration with other scholars

An scholar who influenced Brahmagupta's work was Aryabhata. Brahmagupta's works influences the Arabian mathematician Mohammed ibn Musa al-Khowarizmi. There is no found documentation stating of Brahmagupta collaborating with other scholars.

Historical events that marked Brahmagupta's life.

No particular major events happened around Brahmagupta. Brahmagupta is seen to be mostly unaffected by the events taking place around him. Historical events in the mathematicians geographical area during his/her lifetime.

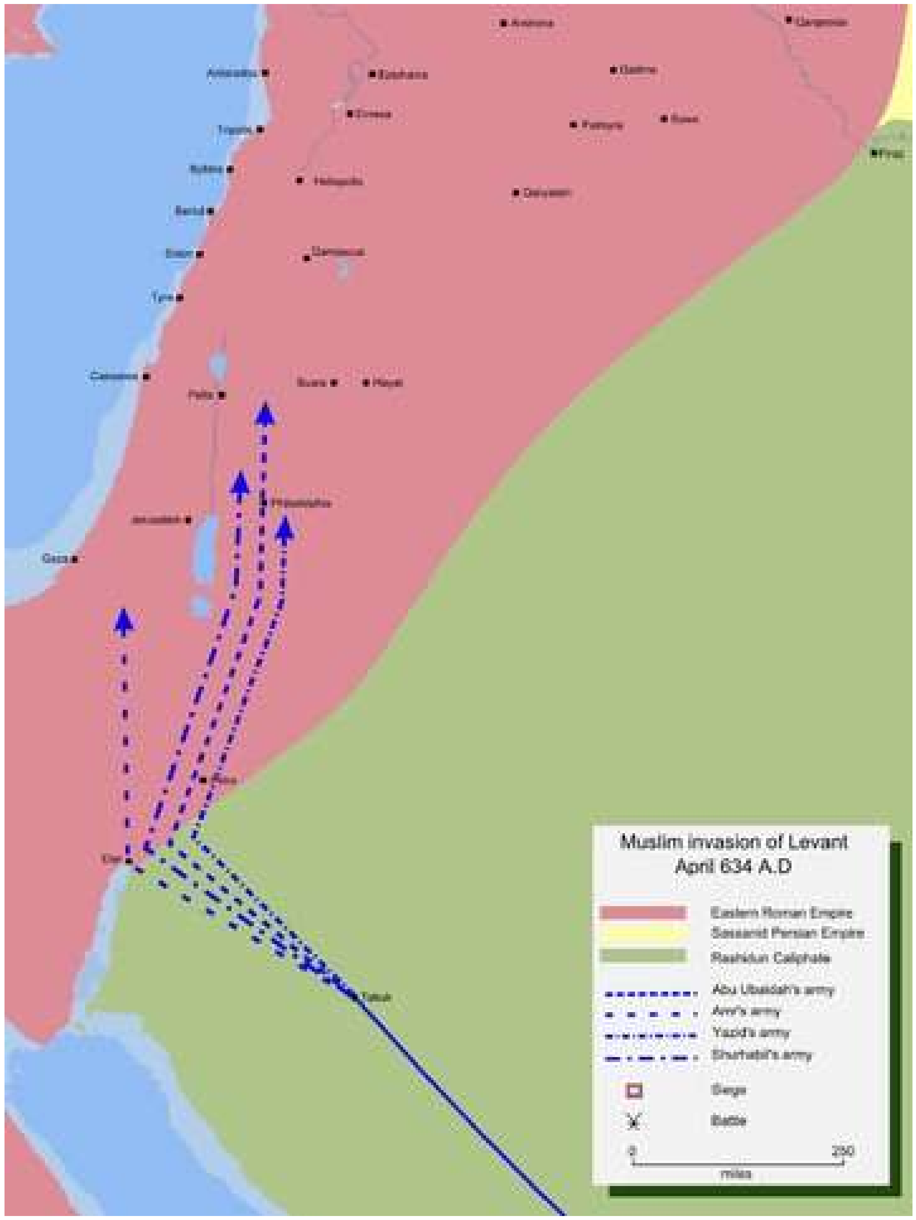
Significant historical events around the world during Brahmagupta's life

The Greek era lasted or about 900 years, ending in 641, when the Arabs burnt down the library at Alexandria. The 1,000 years then on was known as the "Dark Ages" as no more mathematicians were present in the western world. Instead all of the mathematical development shifted toward the Arabs and the Indians.

From 589 to 618 the Sui Dynasty reunites China. Although this formation did not last long, the effects of a bound China did, as it laid a foundation for a booming economy and laid down Buddhism as he main religion in China.

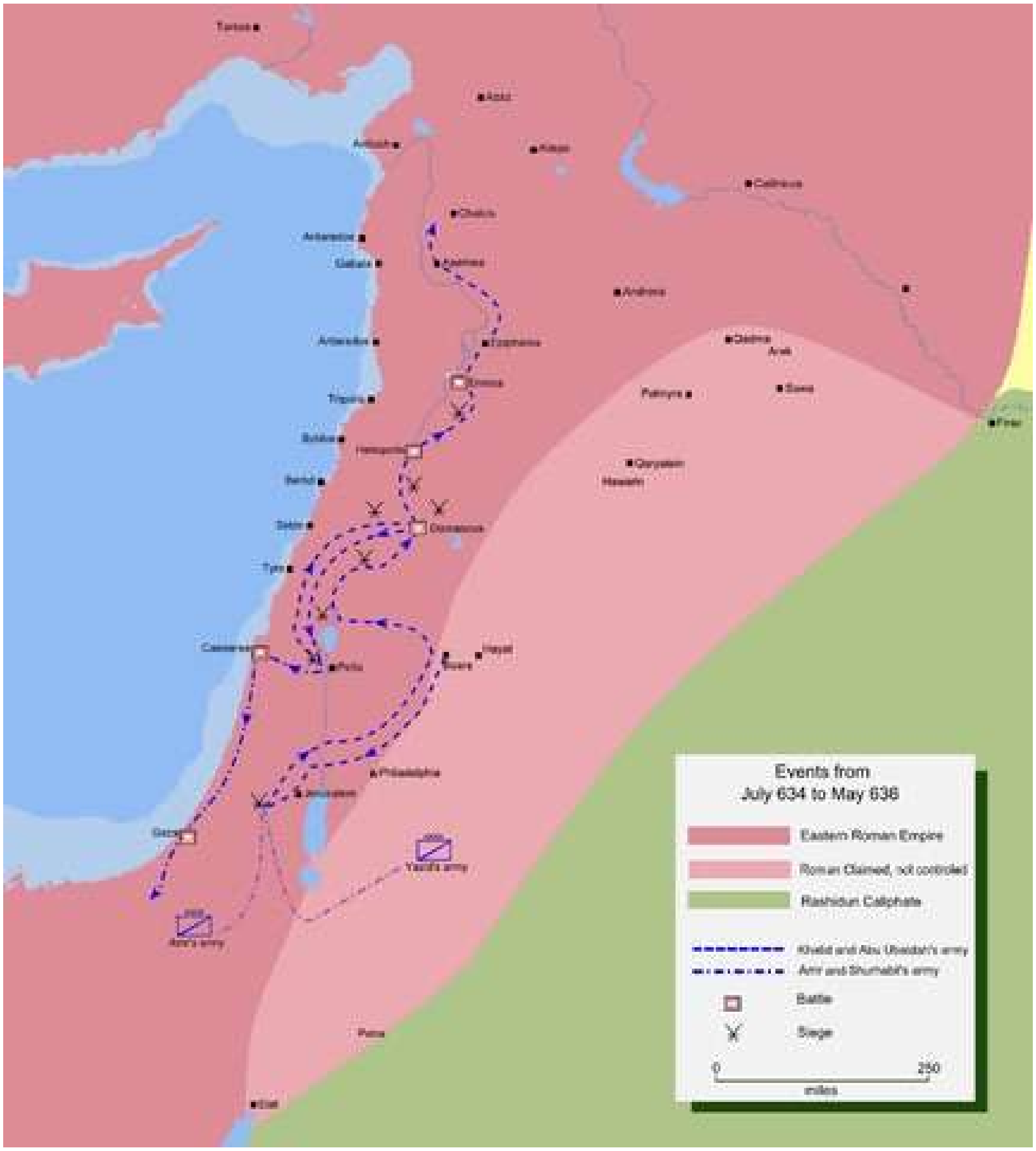
A series of wars was taking place between Byzantine (Eastern Roman) Empire and the Sassanian Empire of Persia it the early stages of Brahmagupta's life. The Byzantine-Sassanian War of 602-628 was the final havoc-filled Sassanian war where Khosrou II of Persia declares war against Phocast and Constantinople. Although the Persians where winning the war during 602 to 622, they failed back with attacks from Heraclius from 622 to 626. As a comeback, the Persians combined forces with the Avars (Eurasian nomads) and the Slavs to try to take Constantinople in 626 for the last attempt of a turn around but failed. With that, Heraclius invaded the Persians land and forced them into ending the war and suing for peace.

With the Byzantine-Sassanian War concluded, both sides were in peace yet vulnerable for others to conquer. This is where the Muslims stepped in and the Islamic Rashidun Caliphate invaded them a few years after. The Islamic Rashidun Caliphate is a successor of Prophet Mohammed (swt), who died in 632. The Muslims defeats the Christian Byzantine army on 30 July 634 in the Battle of Ajnadayn and continued on in their conquering of Syria.



When, Abu Bakr, the first Muslim Caliph and the leader of the Islamic Rashidun Caliphate, died in August of 634, Umar became caliph and slowed down the conquest of Syria. The four caliphs split up to conquer more land. Abu Ubaidah and Khalid went to conquer Emesa, of Roman dissent, and Amr and Shurhabil went towards Palestine.

Undercover, Heraclius of Byzantine rounded up an army to conquer Syria once again. Just in time, the Muslims caught up on this information and prepared for battle as well. The two sides fought in the Battle of Yarmouk in the third week of August, and although the Byzantine army was five times larger than the Muslim's, they lost the battle in October 636.



On going, the Muslims set to conquer Jerusalem. Jerusalem ended up surrendering in April 637. after conquering all of northern Syria, the Muslims conquered Anatolia, modern day Turkey. Then came Egypt, North Africa, and Spain. With the large amount of land and tolerant ruling, many people converted to Islam. And so began the growth of Islam. Even though many converted to Islam, not all residents did. They were not largely discriminated against for not being Muslim, but the Jews, Christians, and Zoroastrians were considered second-class citizens who are protected under Islamic rule.

Significant mathematical progress during the Brahmagupta's lifetime

Another scholar who advanced in mathematics during Brahmagupta's lifetime was Bhaskara I. Hh was a mathematician who was able to establish the Hindu decimal system and approximated the sine function based off of Aryabhata's work. Bhaskara I also contributed to the growth of astronomy and mathematics by writing three books. In 629, he wrote the Aryabhatiya on mathematical astronomy on variable equations and trigonometric functions. One of his remarkable works found within the book was his approximation for sin x.

$$sinx \approx \frac{16x(\pi-x)}{5\pi^2-4x(\pi-x)}, \text{ where } (0 \leq x \leq \frac{\pi}{2})$$

Bhaskara I also furthered on work done by Brahmagupta, such as astronomy and the Pell's Equation.

Connections between history and the development of mathematics

For Brahmagupta, major events in history had nothing to do with him. He only focused on his religion and the progressive movement in astronomy, an important concept for Hinduism. The study of mathematics at the time were of three sections: mathematical sciences, horoscope astrology, and divination.

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